CHAPTER SIX

Simultaneous Equations

- Simultaneous equations may be in the form of two equations, each of which having two unknown values.
- To solve these two equations simultaneously means to determine the values of the two unknown values, which can satisfy each of these two equations.

Q1. Solve the following simultaneously:

$3x + y = 7$

$2x - y = 3$

Soln.

$3x + y = 7 \ldots\ldots\ldots\ldots\ldots\ldots\ldots equation\ (1)$

$2x - y = 3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots equation\ (2)$

1) Consider the sign which appears before each y of the two equations i.e eqn (1) and eqn (2) .

2) Make sure one is positive and the other is negative, as we have in the above given case.

(3) Make sure the values of the y are the same.

4) If these conditions are satisfied, then add the two equations together.

Adding eqn (1) and eqn (2) =>

$3x + y = 7$

$+\underline{2x - y = 3}$

$\underline{5x \quad = 10} \implies 5x = 10, \implies x = \frac{10}{5} = 2.$

$Put\ x = \underline{2}$ into eqn (2)

ie $2x - y = 3,$

$\implies 2(2) - y = 3, \implies 4 - y = 3, \implies 4 - 3 = y,$

$\therefore y = 1.$

Q2. Find the values of x and y which satisfy the equations below simultaneously:

$4x - 2y = 4.$

$2x + 2y = 8.$

Soln.

$4x - 2y = 4$ ……eqn (1)

$2x + 2y = 8$ ……eqn (2).

The signs are opposite i.e one is negative and the other positive. Also the values of y are the same ie. 2y. Add eqn (1) and eqn (2).

$4x - 2y = 4$

$+\underline{2x + 2y = 8}$

$\underline{6x \quad = 12}$

$\Longrightarrow 6x = 12 \Longrightarrow x = \frac{12}{6} = 2, \Longrightarrow x = \underline{\underline{2.}}$

Put $x = 2$ into eqn. (1) i.e. $4x - 2y = 4$, =>

$4(2) - 2y = 4,$

=> $8 - 2y = 4,$

$\therefore 8 - 4 = 2y,$

=> $4 = 2y,$

$$\Longrightarrow 2y = 4 \Longrightarrow y = \frac{4}{2} = 2.$$

$\therefore y = \underline{\underline{2.}}$

Q3. Solve simultaneously these equations:

$x + 2y = 5$

$3x - y = 8$

Soln.

$x + 2y = 5$ ……..eqn (1)

$3x - y = 8$ ……..eqn (2).

Even though the signs are opposite, the values of the y are not the same. In order to make them the same, we multiply equation (2) by 2, $\Longrightarrow 2 \times 3x - 2 \times y = 2 \times 8, \Longrightarrow 6x - 2y = 16$……..eqn (3).

N/B: When we multiply through we get equation (3).

Add eqn. (3) to eqn (1)

(i.e use the other equation we have not multiplied through by the 2) $\Longrightarrow$

$. \quad x + 2y = 5$

$+ \underline{6x - 2y = 16}$

$\underline{7x \quad = 21}$

$$\Longrightarrow 7x = 21, \Longrightarrow x = \frac{21}{7} = 3, \Longrightarrow x = 3.$$

Put $x = 3$ into eqn. (2) i.e $3x - y = 8,$

$\Longrightarrow 3(3) - y = 8, \Longrightarrow 9 - y = 8,$ =>

$9 - 8 = y, \Longrightarrow 1 = y, \Longrightarrow y = \underline{\underline{1.}}$

Q4. Determine the values of *x* and y which satisfy the simultaneous equations

$4x + 3y = 7$

$2x - y = 1$

Soln.

$4x + 3y = 7$……..eqn (1)

$2x - y = 1$……eqn (2)

The signs are opposite but the values of y are not the same. We therefore multiply eqn (2) by 3 i.e

$3 \times 2x - 3 \times y = 3 \times 1, \Longrightarrow 6x - 3y = 3$……eqn (3)

Add eqn. (3) and eqn (1)

$4x + 3y = 7$

$+\underline{6x - 3y = 3}$

$\underline{10x \quad = 10}$

$\Longrightarrow 10x = 10, \Longrightarrow x = \frac{10}{10} = 1.$

Put *x* = 1 into eqn. (1) i.e. $4x + 3y = 7$,

$\Longrightarrow 4(1) + 3y = 7, \Longrightarrow 4 + 3y = 7, \Longrightarrow 3y = 7 - 4 = 3 \Longrightarrow y = \frac{3}{3} = 1.$

Q5. Solve the following simultaneously:

$3x - y = 3$

$4x - y = 5$

Soln.

$3x - y = 3$ ……..eqn (1)

$4x - y = 5$……eqn (2)

Even though the values of y are the same, each one of the signs is negative. One of them must be changed into the positive sign.

Therefore multiply eqn. (1) by – 1

$\Longrightarrow -1 \times 3x - -1 \times y = -1 \times 3,$

$\Longrightarrow -3x + y = -3$……..eqn (3).

Take the other equation i.e eqn. (2) or the one we did not multiply through using the -1 and add it to eqn (3)

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$4x - y = 5$

+-3x + y = -3
x = 2

N/B: 4x + $\bar{3}x$ = x and –y +y = 0.
Put x = 2 into eqn (1) or eqn (2).
Putting x = 2 into eqn (1) ie
$3x - y = 3 \Longrightarrow 3(2) - y = 3, => 6 - y = 3,$
$\Longrightarrow 6 - 3 = y, \therefore y = 3.$
Q6. Find the values of x and y which satisfy simultaneously these equations:
3x – 2y = 2
2x – y = 2

Soln.

3x – 2y = 2 ……eqn (1)
2x – y = 2 ……eqn (2)
N/B: The signs are not opposite and the values of y are also not the same. In order to make one sign positive and the y values the same, we multiply eqn. (2) by -2, i.e
2x – y = 2, $\Longrightarrow \bar{2}$ x 2x – $\bar{2}$ x y = $\bar{2}$ x 2,

$\Longrightarrow$ - 4x + 2y = - 4 ……eqn (3).
Add eqn1 and eqn 3.
3x – 2y = 2
+ -4x +2y = - 4
- x = -2
$\Longrightarrow -x = -2, \Longrightarrow x = 2$ *ie multiply through using* $-1.$
$(ie - 1 \times -x = -1 \times -2 \Longrightarrow x = 2).$
Put x = 2 into either eqn (1) or eqn (2), and using eqn (2) $\Longrightarrow 2x - y = 2, \Longrightarrow$
$2(2) - y = 2, \Longrightarrow 4 - y = 2, \therefore 4 - 2 = y, \Longrightarrow y =$2.

Q7. Solve simultaneously:
x – 4y = -3
2x – y = 1

Soln.

x – 4y = -3……..eqn (1)
2x – y = 1 ……. eqn (2)

Multiply through eqn. (2) using – 4, $\Longrightarrow$

$\overline{4}$ x $2x$ – $\overline{4}$ x y = $\overline{4}$ x 1,

$\Longrightarrow -8x - -4y = -4, \Longrightarrow -8x + 4y = -4$……..eqn (3).

Add eqn (1) and eqn (3)

x – 4y = -3

+-8x +4y = - 4

-7x = -7

$-7x = -7, \Longrightarrow x = \frac{-7}{-7} = 1,$

$\Longrightarrow x = 1. Put\ x = 1\ into\ eqn\ (2)\ i.e\ 2x - y = 1 \Longrightarrow 2(1) - y = 1,$

$=> 2 - y = 1, \Longrightarrow 2 - 1 = y, \Longrightarrow 1 = y \Longrightarrow y =$1.